RADIO ASTRONOMY

2/26 (Wed)

Big Questions by prof's point of view

1) How does the universe work? → cosmology
2) How did we get here? → Galaxy.
3) Are we alone? → Exoplanets

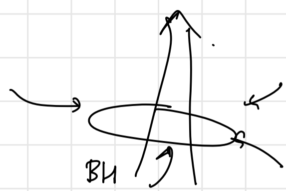
MW picture : (Star – cluster (center)
gas (spiral arm)

H_2 direct → chemical

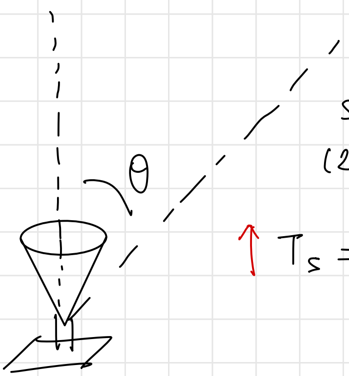
Radio → Antena

3/5 (Wed)

M87 (Virgo cluster) : Jet coming. → SMBH



$z=0 \Rightarrow \sec z = 1$



$\sec z > 1$
$(2.x, 3.x \ldots)$

CGM ⇒ 0.15 ~ 1 B_{500c}

$\uparrow T_s = T_{atm} \cdot \tau_z \cdot \sec(z) \uparrow$

3/12 (수)

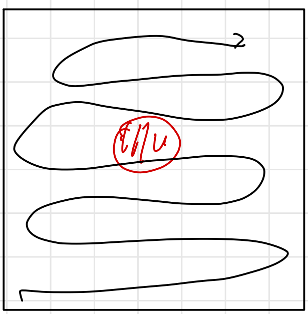
$$\frac{dI_\nu}{ds} = 0 \rightarrow I_\nu = \text{const.} \quad (\text{ray intensity conservation})$$

$$\frac{dI_\nu}{I_\nu} = -\kappa_\nu \, ds \longrightarrow \frac{I_\nu(S_{out})}{I_\nu(S_{in})} = \exp(-\tau)$$

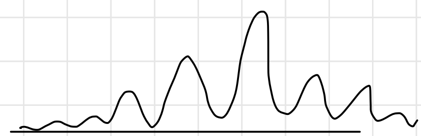
(*) B.B. Radiation.

$$B_\nu(\nu, T) = \frac{2\nu^2 k_B T}{c^2}$$

brightness temperature $\neq$ physical temperature



$\Rightarrow$



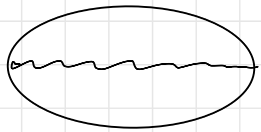
3/17

CMB radiation : ~2.725 K blackbody, evidence of big bang.

1965 discovery

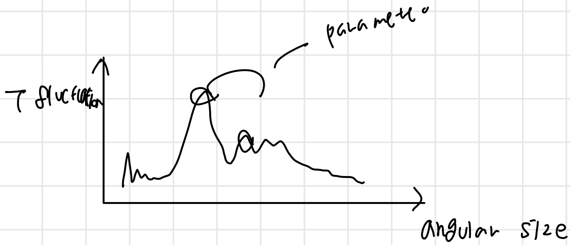
- c/s : cycle per second.

- COBE satellite.



⟶ WMAP ⟶ Planck

· why important?



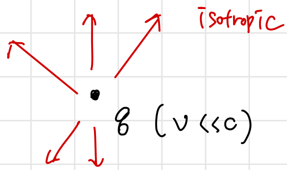
CH III. Radio Telescope and Receivers

Antenna : electromagnetic radiation $\longleftrightarrow$ electrical currents

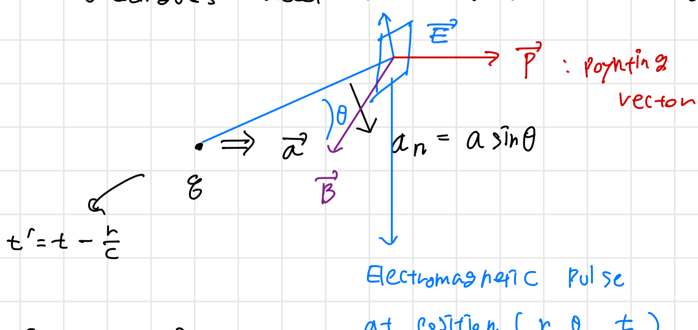
· Polarization : dipole antenna $\lambda \sim L$.

(*) Antenna Fundamentals

· Radiated E vector



· If the "non-relativistic" charge undergoes acceleration: field line distorted.



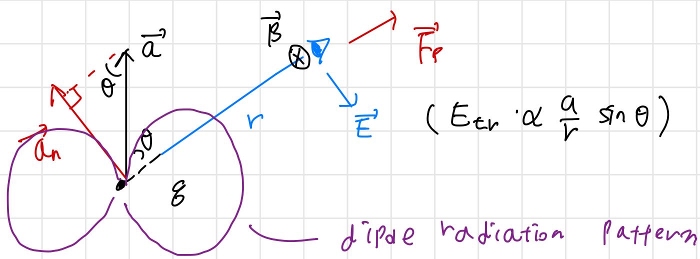
$\vec{E}$: directly opposite to $\vec{a}$ (positive q)
amplitude $\sim \frac{1}{r}$
energy flux $\sim \frac{1}{r^2}$ $(\propto |E|^2)$

$$E_{tr} \propto \frac{q a_n}{r} = \left(\frac{qa}{r}\right) \sin\theta .$$

$$\boxed{\therefore \ \vec{E}(r,t) = E_{tr} \cdot \hat{n} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q\,a(t')\sin\theta}{c^2 r} \cdot \hat{n} = \frac{q}{4\pi\varepsilon_0 c^2 r} (\vec{a} \times \hat{k}) \times \hat{k}}$$

[V/m]

"Transverse E vector"



- Energy flux density

$\vec{F}$ [w/m^2] : depends on $\vec{E}$ & $\vec{B}$

- Energy density of $\vec{E}$ & $\vec{B}$

$$\frac{\varepsilon_0 E^2}{2} \qquad \frac{B^2}{2\mu_0}$$

where $B = \frac{E}{C}$

$$\therefore \vec{F_p} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) \quad [w/m^2]$$

$(\vec{S})$

$$C = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

$$|\vec{F_p}| = \varepsilon_0 C \cdot \vec{E} \cdot \vec{E} = \varepsilon_0 C E_{tr}^2.$$

$$\therefore |\vec{F_p}(r, \theta, t)| = \frac{q^2 \sin^2\theta \, a^2(t')}{(4\pi)^2 \varepsilon_0 C^3} \frac{1}{r^2} \quad [W/m^2]$$

(*) Larmor formula : Total power radiated by single charge into all direction.

$$\Rightarrow P(t') = \int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} F_p(r, \theta, t) \, r^2 \sin\theta \, d\varphi \, d\theta = \frac{1}{6\pi\varepsilon_0} \frac{q^2 a^2(t')}{C^3} \quad [W]$$

$\rightarrow P(t)$ (reference of time)

$a^2(t)$

instantaneous power

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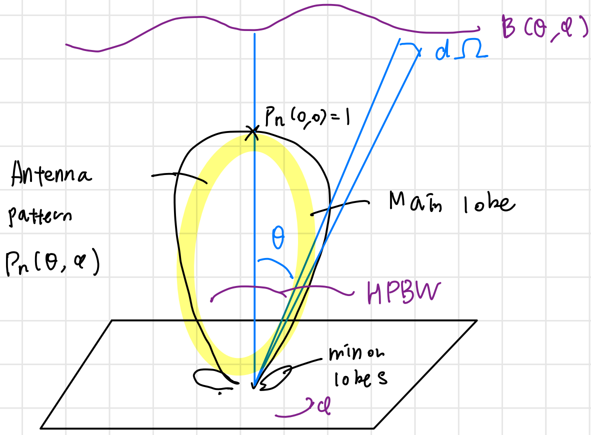
< Antenna Fundamentals >



(effective aperture (A_e))

· Antenna : EM radiation

$\leftrightarrow$ Electric Currents

· Power "Gain" $G(\theta, \varphi)$

= the power transmitted per unit solid angle in direction (θ, φ)

(relative to an isotropic antenna)

cf) $G\,[dB] = 10 \log_{10} G$.

For isotropic lossless antenna $\langle G \rangle = 1$

$P(\theta, \varphi)$:= Power per unit solid angle radiated in (θ, φ)

$$G(\theta, \varphi) = \frac{P(\theta, \varphi)}{\frac{1}{4\pi} \iint P(\theta, \varphi)\, d\Omega}$$

$\rightarrow$ directive gain, or normalized power pattern

· Good approximation in Short dipole antenna : $P \propto \sin^2\theta$

$\Rightarrow$ Directive gain $G(\theta, \varphi) = \frac{3}{2} \sin^2\theta$

(*) Normalized Power pattern

$$P_n(\theta, \varphi) = \frac{1}{P_{max}} \cdot P(\theta, \varphi)$$

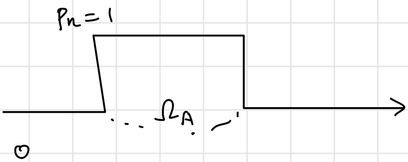
- Beam / main beam solid angle

$$\Omega_A = \iint_{4\pi} P_n(\theta, \varphi)\, d\Omega = \int_0^{2\pi}\int_0^{\pi} P_n(\theta, \varphi) \sin\theta \, d\theta \, d\varphi \quad [Sr]$$

→ Beam solid angle

" Solid angle of an ideal antenna"

$P_n = 1$ (for all Ω_A) & $P_n = 0$ everywhere else



$$\Omega_{MB} = \iint_{\substack{\text{main} \\ \text{lobe}}} P_n(\theta, \varphi)\, d\Omega.$$

→ main beam solid angle

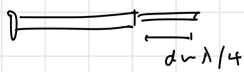
3/26 (4)

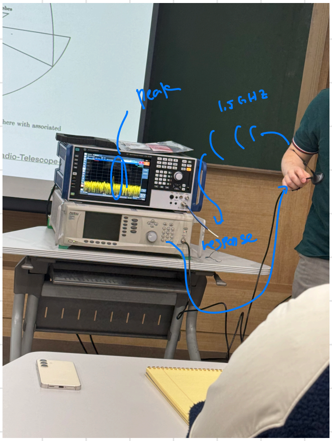
$1.5 \; 10^9$

$\nu = 1.5\,GHz$

$\Rightarrow \lambda = 2 \cdot 10^{-1}$ m.

Signal generator and Oscilloscope.





1.5 GHz Peak

cf) Small peak at 1.485 GHz → HD-SDI signal

→ RFI_ radio frequency interference

$$\Omega_A = \iint_{4\pi} P_n(\theta, \varphi)\, d\Omega = \text{Beam solid angle}$$

$$\Omega_{MB} = \iint_{\substack{\text{main} \\ \text{lobe}}} P_n(\theta, \varphi)\, d\Omega = \text{main beam solid angle}$$

$$\eta_B = \frac{\Omega_{MB}}{\Omega_A} = \text{main beam efficiency}$$

→ ideal case: no minor lobe, $\eta_B = 1$.

· Half power beam width (HPBW)

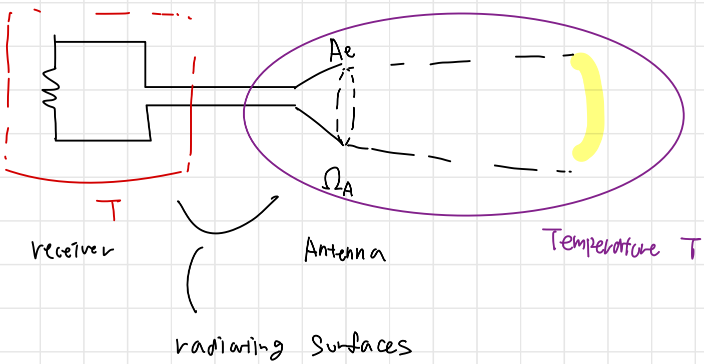
· FWHP $\sim \frac{\lambda}{D}$

· Effective aperture $A_e = \frac{P_e}{|\langle \vec{S} \rangle|}$

$\hookrightarrow$ Power density "flux" $[W/m^{-2}]$

· Geometric aperture is A_g. (if given)

$\Rightarrow \eta_A = \frac{A_e}{A_g}$ (aperture efficiency)



P_ν (spectral power) from all directions

Ae: θ φ dependence

polarization

$$\Rightarrow P_\nu = \frac{1}{2}\int_{4\pi} A(\theta,\varphi) B_\nu \, d\Omega$$

??

Spectral power, unpolarized BBR

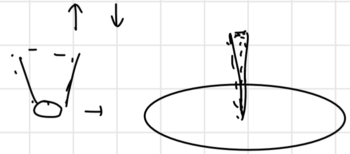
$$k_B T = \frac{1}{2}\int_{4\pi} A(\theta,\varphi) \frac{2k_B T}{\lambda^2} \, d\Omega$$

Note that $\frac{1}{4\pi}\int_{4\pi} A(\theta,\varphi) \, d\Omega = \langle A_e \rangle$

$= \frac{\lambda^2}{4\pi}$ (???)

$$\Omega_A \equiv \int_{4\pi} \frac{A(\theta,\varphi)}{A_e} \, d\Omega$$

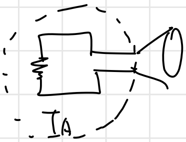
$$\rightarrow \boxed{A_e \Omega_A = \lambda^2}$$



(*) Antenna Temperature T_A

$$T_A \equiv \frac{P_\nu}{k_B}$$ → spectral power that antenna measures

$T_A = 1\,K \Rightarrow P_\nu = 1.38 \times 10^{-23}\ W \cdot Hz^{-1}$



↳ BBR with known T

$$T_A = \frac{1}{2}\frac{1}{k_B} \cdot [S_\nu A_e]$$

A_e: effective area $[m^2]$

S_ν: flux density $[W \cdot m^{-2} Hz^{-1}]$

$$A_e = \frac{2 k_B T_A}{S_\nu}$$

$\left(\begin{array}{l} 1 Jy = 10^{-26}\ [W \cdot m^{-2} \cdot Hz^{-1}] \\ T_A = 1K \end{array} \right.$

$\rightarrow A_e = 2761\,m^2.$

3/31 (월)

(*) Two def of T

1) Brightness temperature := given brightness, $T_b = \dfrac{c^2}{2k_B\nu^2} I_\nu$ ← Source

2) Antenna temperature := ideal resistor temp $T_A = \dfrac{P_\nu}{k_B}$ ← measured (antenna)

In an arbitary radiation field

$I_\nu(\theta,\varphi)$ ... specific intensity $[W \cdot m^{-2}\, sr^{-1}\, Hz^{-1}]$ → $T_b \sim I_\nu$

Moreover, output power of antenna can be introduced as:

$$P_\nu = \frac{1}{2}\int_{4\pi} \underbrace{A(\theta,\varphi)}_{A_e P_n(\theta,\varphi)} I_\nu(\theta,\varphi)\, d\Omega \longrightarrow T_A = \frac{P_A}{k_B}$$

$A_e P_n(\theta,\varphi)$, Antenna response

$$\boxed{\therefore\ T_A = \frac{1}{\lambda^2}\cdot\int_{4\pi} A(\theta,\varphi)\, T_b(\theta,\varphi)\, d\Omega}$$ → Convolution!!

특징 = $\int (\ ,\ ) \cdot (\ ,\ )$

장비 성질 천체 intrinsic property

(*) 2 extreme cases

-(i) Very expended source (nearly constant T_b)

$T_b(I_\nu)$

$$\Rightarrow T_A = \frac{T_b}{\lambda^2}\int_{4\pi} A(\theta,\varphi)\, d\Omega$$

$\lambda^2 = A_e \Omega_A$

$$\boxed{T_A = T_b}$$

ex) CMB pattern

$T_b \sim 2.73K$.

-(17) Compact object

= A compact source covering Ω_s with T_b (= const)

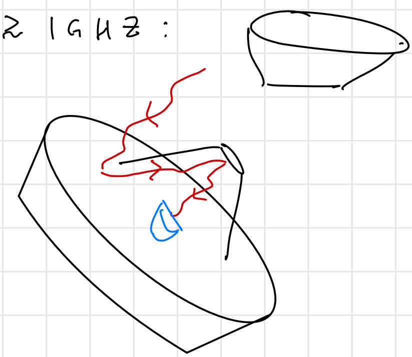
($\Omega_s \leq \Omega_A$)

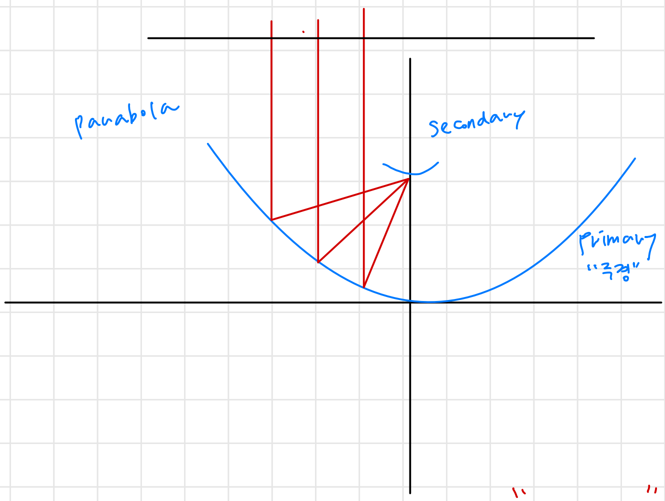
$$\rightsquigarrow \quad T_A \simeq \frac{1}{\lambda^2} A_e T_b \Omega_s$$

$$\boxed{\therefore \ \frac{T_A}{T_b} = \left(\frac{\Omega_s}{\Omega_A}\right)}$$

Reflector Antennas → Antennas for Radio Astronomy
(Large "reflectors")

$\nu \gtrsim 1\,\mathrm{GHz}$: useful



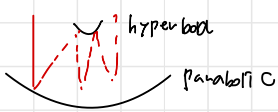


$$z = \frac{r^2}{4f}$$

focal ratio $\frac{f}{D}$

⇒ plane wave to "single focus"
with same path length

· Optical configuration

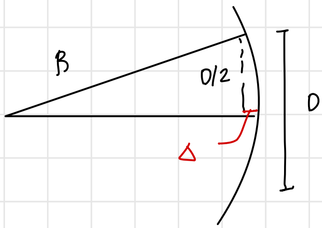
① 

⇒ (Cassegrain)

② 

⇒ Gregorian

Far-field distance
(parallel light)



$\Delta < \frac{\lambda}{16}$ $\Rightarrow R^2 = (R-\Delta)^2 + \left(\frac{D}{2}\right)^2$

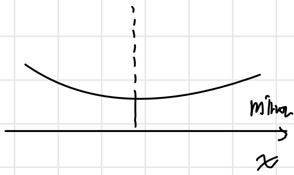
$\Rightarrow R \sim \frac{2D^2}{\lambda}$

→ Not that need to worry for Astronomical object.

· In the far-field,

the electric field pattern (of an aperture)

$$f(l) = \int_{\text{apperture}} \underbrace{g(u)}_{\text{electric field distribution}} e^{-2\pi i l u} \, du \qquad \left(u \equiv \frac{x}{\lambda}\right)$$



cf. Fraunhofer diffraction, Airy disk

4/2 (금)

Radiometers

- Heterodyne Receivers → ν information keep, Spectroscopic obs
- Bolometric Receivers → 온도계. photon → ΔT.
  → ν wide range

· Quantum efficiency ↑

· Noise ↓

· Dynamic rang ↑

$$\boxed{\Rightarrow \sigma = \frac{T_{sys}}{\sqrt{\Delta\nu \cdot \underbrace{\tau}_{\text{time}}}}}$$

· Number, physical size of pixels

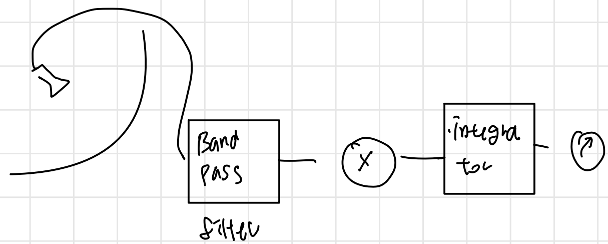
· Time response

· Spectral response · Spectral response

$$T_{\text{계}} = \underbrace{T_{CMB} + T_{rsb} + \cdots}\ + \Delta T_{source}$$

$\to$ multi-wavelength obs, even in radio window

· Square-law detector . $\otimes$



$$\Downarrow$$

$$\nu_{BF} - \Delta\nu/2 \le \nu \le \nu_{BF} + \Delta\nu/2$$

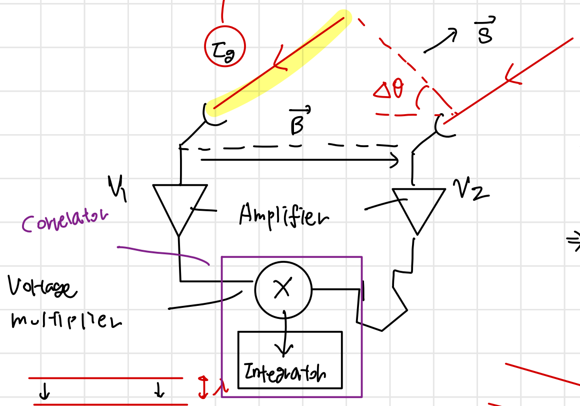
4/23

$: \tau_g = \dfrac{\vec{B}\cdot\vec{S}}{c}$

$\vec{B}$ = baseline (기선)

(*) Two element Interferometers.

geometric delay



$\to V_2 = V\cos(\omega(t-\tau_g))$ $\quad \in e^{i\omega(t-\tau_g)}$

$V_1 = V\cos(\omega t)$ $\quad \in e^{i\omega t}$

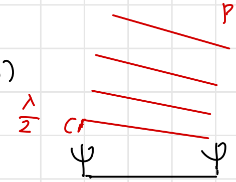
$\Rightarrow V_1V_2 = \dfrac{V^2}{2}\left[\cos(2\omega t - \omega\tau_g) + \cos(\omega\tau_g)\right]$

$$\Rightarrow \boxed{\begin{aligned} R = \langle V_1V_2\rangle &= \frac{V^2}{2}\cos(\omega\tau_g) \\ &= \frac{V^2}{2}\cos\left[\frac{2\pi}{\lambda}\vec{B}\cdot\vec{S}\right] \end{aligned}}$$ ... Source Signal

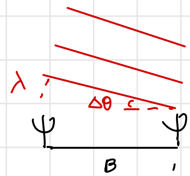
Plane wave

i)

ii)



iii)



· Phase-matched.

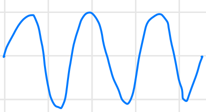
⇒ Constructive interference

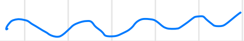
· phase 반대

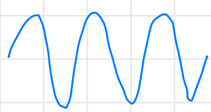
⇒ destructive interference

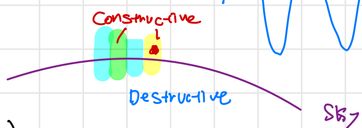
· Again, constructive.

$V_1 + V_2$ output





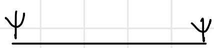


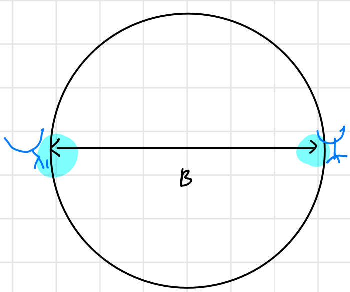


$\Rightarrow \Delta\theta \sim \dfrac{\lambda}{B}$.

$\left[\begin{array}{ll} B\downarrow & \Delta\theta\uparrow \text{ (resolving power}\downarrow) \\ B\uparrow & \Delta\theta\downarrow \text{ (" }\uparrow) \end{array}\right.$

$\Delta\theta \sim \dfrac{\lambda}{B}$.





Angular resolution

$\theta \sim \dfrac{\lambda}{B}$

1) 우선 나머지 (///) 영역 비었다고 가정

→ 거울면에 부분 조각 2개짜리.

4/28

(*) Fourier Series and Transforms

- Temporal & spatial "Periodicity" → In time : measured by ν [Hz]
  → In space : wavelength λ [m]

- Fourier transform Pairs

$$\mathcal{F}\{f(t)\}(s) = F(s) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi st}\, dt$$

↓ inverse FT

$$F^{-1}\{F(s)\} = f(t) = \int_{-\infty}^{\infty} F(s)\, e^{j2\pi st}\, ds$$

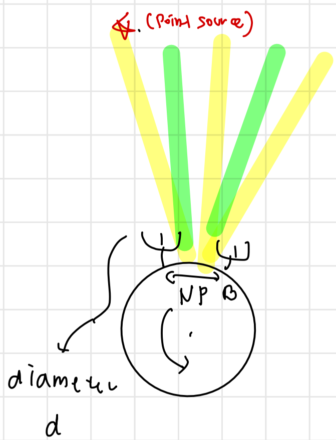
Two element interferometer

- "Correlation" $\quad B(\tau) \propto \frac{E^2}{T} \int_0^T e^{i\omega t} e^{-i\omega(t-\tau)}\, dt$

$$\xrightarrow[\tau \gg \frac{2\pi}{\omega}]{} B(\tau) = \frac{\omega}{2\pi} E^2 \int_0^{2\pi/\omega} e^{i\omega\tau}\, dt \propto E^2 e^{i\omega\tau}$$

↳ varies with "τ" → Mutual coherence function

(*) Interference band



ν = observing frequency = 6GHz

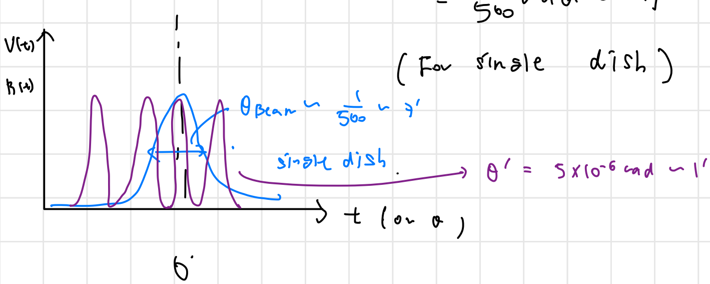
B = 10km

d = 25m

$\Rightarrow \theta_{beam} \sim \frac{\lambda}{d}$ → 6 GHz, ↳ 25 m

$= \frac{1}{500}$ rad $\sim 7'$

(For single dish)



$$B(T) \propto E^2 e^{i\omega\tau} \rightarrow E^2 e^{i\left(\frac{2\pi}{\lambda}\vec{B}\cdot\hat{s}\right)}$$

$\therefore \theta' = \frac{\lambda}{B} = 5\times10^{-6}$ rad $\sim 1''$

$I_\nu(\vec{s})$: Radio Source distribution on the sky

(*) 2 element Still ...

- The power received per $d\nu$ from the source element $d\Omega$

$\Rightarrow$ · $A(\vec{s})\, I_\nu(\vec{s})\, d\Omega\, d\nu$ → Output of Correlator $R = A(\vec{s}) I_\nu(\vec{s}) e^{i\omega\tau} d\Omega d\nu$

Effective collecting area (for dish)

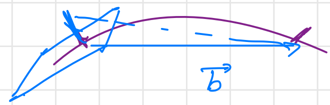
Total response $\quad R(\vec{B}) = \iint_\Omega A(\vec{s}) I_\nu(\vec{s}) \exp\left[i\, 2\pi\nu \left(\frac{1}{c}\vec{B}\cdot\vec{s}\right)\right] d\Omega\, d\nu$

Source distribution

↳ visibility function

(*) Interferometry (of 2 element)

$$R(\tau) \propto \frac{\omega}{2\pi} E^2 \int_0^{2\pi/\omega} e^{i\omega\tau}\, dt$$

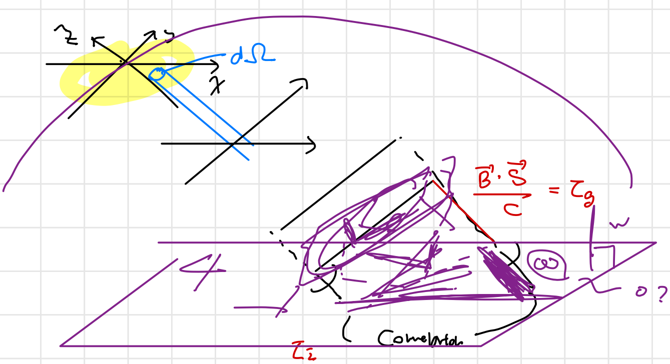


$A(\vec{s})\, I_\nu(\vec{s})\, d\Omega\, d\nu$

Visibility function $\quad R(\vec{B}) = \iint_\Omega A(\vec{s})\, I_\nu(\vec{s}) \exp\left[i\, 2\pi\nu\left(\frac{1}{c}\vec{B}\cdot\vec{s} - \tau_i\right)\right] d\Omega\, d\nu$

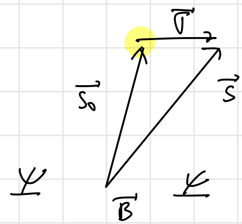
Measurement

↳ What we want to derive



$\frac{d\vec{\sigma}}{\nu^2} \cdot \frac{1}{\nu^2}$

$\vec{S} = \vec{S}_0 + \vec{\sigma}$



V. F $\quad R(\vec{B}) = \iint_\Omega A(\vec{s}_0 + \vec{\sigma})\, I_\nu(\vec{s}_0 + \vec{\sigma}) \exp\left[i\, 2\pi\nu\left(\frac{\vec{B}\cdot\vec{S}}{c} - \tau_i\right)\right] d\Omega\, d\nu$

$$= \underbrace{\exp\left[i\cdot 2\pi\nu\left(\frac{\vec{B}\cdot\vec{S}}{c} - \tau_i\right)\right] d\nu}_{\text{phase term}} \underbrace{\iint A(\vec{\sigma})\, I(\vec{\sigma}) \exp\left[i\cdot 2\pi\nu \cdot \frac{\vec{B}\cdot\vec{\sigma}}{c}\right] d\vec{\sigma}}_{\text{Complex visibility } V(B)}$$

$$V(B) := \iint A(\vec{r})\, I(\vec{r})\, \exp\left[i \cdot 2\pi \cdot \frac{\vec{B}}{\lambda} \cdot \vec{r} \right] d\vec{r}$$

claim : $\frac{\vec{B}}{\lambda} = (u, v, w)$ 3D 확장.

(*) visibility

$$V(u,v,w) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(x,y)\, I(x,y)\, \exp\left[i \cdot 2\pi \left(ux + vy + w\sqrt{1-x^2-y^2} \right) \right] \cdot \underbrace{\frac{dx\,dy}{\sqrt{1-x^2-y^2}}}_{\sim 1}$$

$$\underbrace{V(u,v,w) \cdot e^{-2\pi i w}}_{\simeq V(u,v,0)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(x,y)\, I(x,y)\, e^{i 2\pi (ux+vy)}\, dx\,dy$$

$$\therefore \text{visibility} \quad V(u,v,0) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \boxed{A(x,y)\, I(x,y)}\, e^{2\pi i (ux+vy)}\, dx\,dy$$

→ F.T.

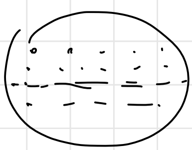
↓ Inverse F.T.

$$\int_{-\infty}^{\infty} V(u,v,0)\, e^{-2\pi i (ux+vy)}\, du\,dv = A(x,y) \cdot \text{"}I(x,y)\text{"}$$

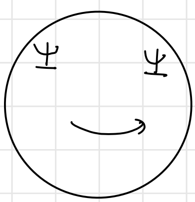
· To maximize (or encode many information for) u, v

$\frac{\vec{B}}{\lambda}\uparrow = (u, v, w)$

- B ↑ or separation ↓ (pixel ↑)
- Earth rotation



1 dish



$N \longrightarrow N(N-1)/2$

2   1

⋮

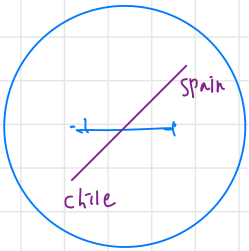
8

⋮

60

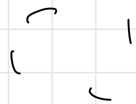
$V(0,0) = ?$ ⟶ $\vec{B} \simeq 0$ seem meaningless but

"Total Flux density!"



NS
SW

Antenna N 개 $\Rightarrow$ $\underline{{}_N C_2 \times 2}$ (u, v) pair.

$\Rightarrow$ (Time hour survey) $\longrightarrow$  uv plane 채우기.

$B \sim 10000\,km$ $\quad \lambda = 1.3\,mm \sim 1\,mm$

$$\theta \sim \frac{\lambda}{B} = \frac{10^{-3}}{10^{7}} = 10^{-10}\,rad \xrightarrow[\times 206265]{} 2 \times 10^{-5}\,as \quad \boxed{20\,\mu as}$$

$$= \frac{180}{\pi}\ 60\ 60$$

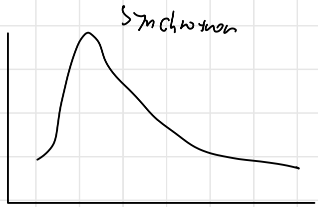
BH
- Blue : 동서 대칭
- red : 남북 비대칭.

5/12

EHT

Black hole shadow : $\sqrt{27} \cdot \frac{GM}{c^2}$.

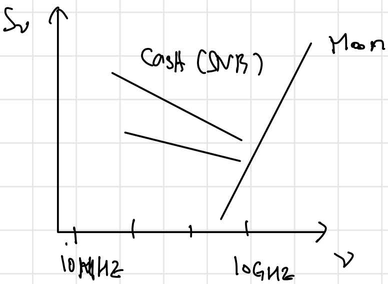
· θ : Sgr A*, M 87*.



$\nu \uparrow, \lambda \downarrow \Rightarrow$ Inner structure, Img. Center of SMBH.

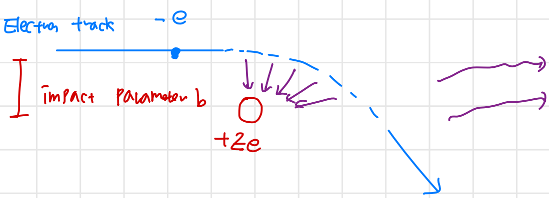
CH5. Astrophysics Process

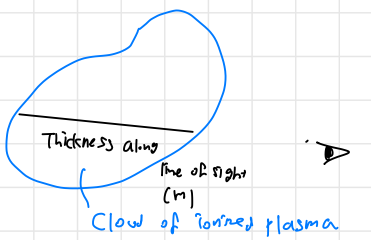
The nature of Radio sources



(*) Bremsstrahlung radiation

· High, ionized gas (plasma) : ← GPT. : $T \gtrsim 10^5$ K. (10^6 K)





· Maxwell-Boltzmann distribution of e. (Thermal equilibrium)

$$\left\langle \frac{1}{2} m v^2 \right\rangle = \frac{3}{2} k_B T$$

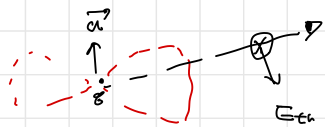
· Completely ionized [electron $-e$; ions of $+Ze$

· non-relativistic ($v \ll c$)

· Poynting vector $\vec{F_p} := \dfrac{\vec{E} \times \vec{B}}{\mu_0}$ [$W m^{-2}$]

$$\left| \vec{F_p}(r,\theta,t) \right| = \frac{e^2 \sin^2\theta \, a^2(t')}{(4\pi)^2 \varepsilon_0 c^3 r^2} \quad [W.m^{-2}] \longrightarrow$$

(*) Larmor formula : Total power radiated.



$E_{th} \propto \dfrac{a}{r} \sin\theta$

$$P(t) = \int_0^{\pi} \int_{\varphi=0}^{2\pi} F_p(r,\theta,t) r^2 \sin\theta \, d\varphi \, d\theta$$

$$= \frac{1}{6\pi\varepsilon_0} \cdot \frac{e^2 a^2(t)}{c^3} \quad [W]$$

(i) Radiative E emitted by a single electron

$$\vec{a} = \frac{\vec{F}}{m} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2}{r^2}\cdot\frac{1}{m}\cdot\hat{r}$$

$$\boxed{a_{max} \simeq \frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2}{mb^2}}$$

조사 1

· Total E emitted

$$Q(b,v) = \int_{-\infty}^{\infty} P(t)\,dt = \frac{1}{6\pi\varepsilon_0}\cdot\frac{e^2}{c^3}\underline{\int_{-\infty}^{\infty} a(t)^2\,dt}.$$

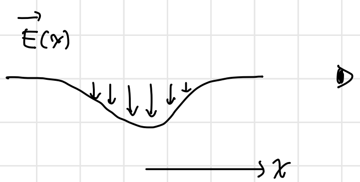
$\simeq a_{max}\cdot\tau_b$ (τ_b = collision time $\sim \frac{b}{v}$)

조사 2

$$Q(b,v) \simeq \frac{1}{6\pi\varepsilon_0}\cdot\frac{e^2}{c^3}\cdot a_{max}^2\,\tau_b$$

$$= \frac{1}{(4\pi\varepsilon_0)^2}\cdot\frac{2}{3}\cdot\frac{Z^2e^6}{c^3m^2b^3v}$$

(ii) The "freq" of the emitted radiation.



$$\tau_b = \frac{b}{v}$$

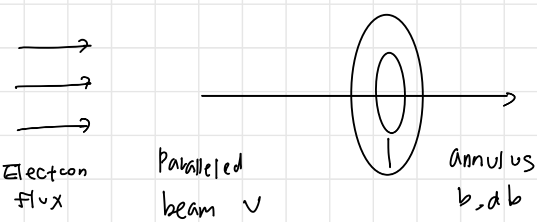
$$\omega \sim \frac{1}{\tau_b} \simeq \frac{v}{b}$$

characteristic freq

$$\nu = \frac{\omega}{2\pi} \simeq \frac{v}{2\pi b}$$

$$b = \frac{v}{2\pi\nu} \quad : db = -\frac{v}{2\pi\nu^2}\,d\nu$$

(iii) "Thermal" electron and a single ion.



electron flux

$$\mathbb{P}_b(b,v)\,db = Q(b,v)\,n_e v\,2\pi b\,db$$

$$= \frac{1}{(4\pi\varepsilon_0)^3}\cdot\frac{2}{3}\cdot\frac{Z^2e^6}{c^3m^2b^3v}$$

$$\mathbb{P}(b,v) \longrightarrow \mathbb{P}(\nu,v) \qquad \left(db = -\frac{v}{2\pi\nu^2}\,d\nu\right)$$

$$\Rightarrow \int_{b_1}^{b_2}\mathbb{P}_b(b,v)\,db = -\int_{\nu_1}^{\nu_2}\mathbb{P}_\nu(\nu,v)\,d\nu$$

in $d\nu$ at ν

$$\Rightarrow \mathbb{P}_\nu(\nu,v) = -\mathbb{P}_b(b,v)\frac{db}{d\nu} = Q(b,v)\,n_e v\,2\pi b\,\frac{v}{2\pi\nu^2} = \frac{1}{(4\pi\varepsilon_0)^3}\cdot\frac{8\pi^2}{3}\,n_e\,\frac{Z^2e^6}{c^3m^2v}\,d\nu$$

[W/ion]

$\frac{1}{2} m v^2 = h \nu_{max} \longrightarrow \nu_{max} = \frac{m v^2}{2h}$ (classical limit)

Cutoff freq which radiation $\to 0$

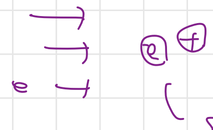
$$\langle P_\nu(\nu) \rangle_{ion} = \int_{v_{min}}^{\infty} P_\nu(\nu, v)\, P(\vec{v})\, 4\pi v^2 dv \quad [W \cdot ion^{-1} \cdot Hz^{-1}]$$

$v_{min} \hookrightarrow = \sqrt{\frac{2h\nu}{m}}$

- Volume emissivity ; $j_\nu(\nu) d\nu = n_i \cdot \langle P_\nu(\nu) \rangle_{ion} d\nu$

$\hookrightarrow$ ion density

$$\xrightarrow[\text{Integration}]{} j_\nu(\nu) d\nu = g(\nu, T, Z) \frac{1}{(4\pi\varepsilon_0)^3} \cdot \frac{32}{3} \left(\frac{2}{3} \frac{\pi^3}{m^3 k_B} \right)^{1/2} \frac{Z e^6}{c^3} n_e n_i \frac{e^{-h\nu/k_B T}}{T^{1/2}} d\nu$$



screening effect.

$\Rightarrow g$ account.

2/5/28 (7)

Review (6/4)

Gravitational neutrino $\rightarrow$ new probe rather than EM wave.

$$\dot{\Delta} = \frac{T_{s70}}{\sqrt{6\nu\tau}}$$

3.7. Radiometers

:= measure the average power of the noise coming from a radio telescope in ν domain

· $N = \Delta\nu \cdot \tau$

(*) Band-limited noise.

$P_\nu = kT$ [열역학적 대략 상 noise power] $\longrightarrow T_N = \frac{P_\nu}{k}$

(low ν limit)

관측 천체

system noise temp $= T_{sys}$ = total noise power $= T_{cmb} + T_{rsb} + \Delta T_{source} + [1 - \exp(-\tau_A)] T_{atm} + T_{spill} + T_{r} \cdots$

백그라운드

내부 잡음.

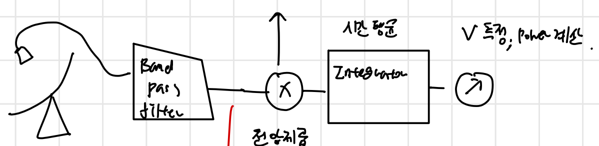
(*) Radiometer

$\Rightarrow \quad P = k_B T_{sys} \Delta\nu$ 측정하는 장치

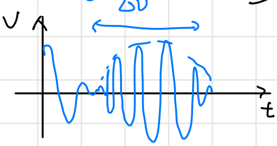
square law detector

시간 평균

V 측정; Power 계산.



$\tau \sim \frac{1}{\Delta\nu}$



전압제곱

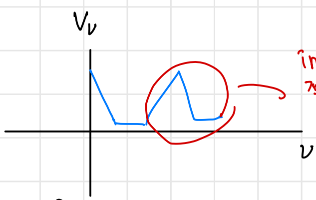
→ Power

→ 평균 0, power ~ V^2

V_{RF} = 중심주파수,

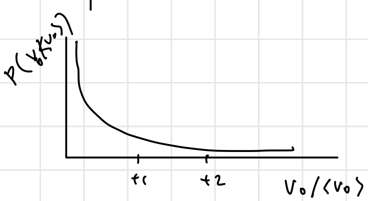
$V_{out} \propto V^2 = \frac{V_0^2}{2} \cos^2(2\pi\nu_{RF} t)$

$= \underbrace{\frac{V_0^2}{2}}_{DC} [1 + \underbrace{\cos 4\pi\nu_{RF} t}_{AC}]$



integration 적분 과정 시나리오

$N = 2\Delta\nu\tau$.



$\Rightarrow \sigma_T = \frac{\sigma_p}{\sqrt{N}} = \frac{\sqrt{2} \cdot T_s}{\sqrt{2\Delta\nu\tau}} = \frac{T_s}{\sqrt{\Delta\nu\tau}} \quad (\Delta T \sim O(1) \cdot \sigma_T)$

Fourier Transform

$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |F(s)|^2 ds$$

- Continuous F.T.
- Discrete F.T. → FFT

$$F(s) \equiv \int_{-\infty}^{\infty} f(x)\, e^{-2\pi i s x}\, dx \quad \text{(Forward)}$$

$$f(x) \equiv \int F(s)\, e^{2\pi i x s}\, ds \quad \text{(inverse)}$$

$$F(s) \Leftrightarrow f(x)$$

→ $\overline{F(s)} \cdot F(s) = |F(s)|^2$: power spectrum.

$\{e^{i\phi} = \cos\phi + i\sin\phi\}$ → Complete, orthogonal set.

↳ Eigenfunction of differential eq.

A.2. Discrete F.T.

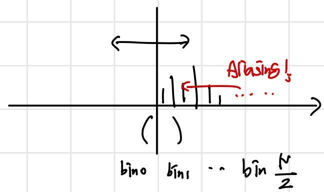
Uniformly sampled $x_0, x_1, \cdots, x_{N-1}$ ⇒ $X_k \equiv \sum_{j=0}^{N-1} x_j \cdot e^{-2\pi i j k/N}$ → $A_k \cdot e^{i\phi_k}$.

$$x_j \equiv \frac{1}{N} \sum_{k=0}^{N} X_k\, e^{2\pi i k j/N}$$

(*) Sampling thm.

$$X_{-k} = \overline{X_k}$$

Sampling spacing $\Delta t \le \frac{1}{2\Delta\nu}$ → perfect 재구성 ok.



→ Samping 주파수는 원래 신호 최대 주파수의 2배 이상이어야 신호 복원 가능.

$$\frac{N}{2} - \frac{f_s}{2} = \frac{1}{2\Delta t}$$

baseband: $[0 \sim \nu]$ $\xrightarrow{\text{in real}}$ $[100, 101]$ → baseband heterodyning
↳ aliasing 이용.

$0 \sim \frac{f_s}{2}$: 1st Nyquist zone $\frac{f_s}{2} \sim f_s$: 2nd Nyquist zone, ⋯

① $f(x) + g(x) \Leftrightarrow F(s) + G(s)$ $af(x) \Leftrightarrow aF(s)$

② $f(x-a) \Leftrightarrow e^{-2\pi i a s} F(s)$ ③ $f(ax) = \frac{f(s/a)}{|a|}$

③ $f(x) \cdot \cos(2\pi\nu x) \Leftrightarrow \frac{1}{2} F(s-\nu) + \frac{1}{2} F(s+\nu)$ ⑤ $\frac{df}{dx} \Leftrightarrow i2\pi s F(s)$

(*) Convolution,

$$h(x) = f * g = \int_{-\infty}^{\infty} f(u)\, g(x-u)\, du.$$ $\ni$ $f * g \Leftrightarrow F \cdot G.$

cf) Cross-Correlation

$$f \star g = \int_{-\infty}^{\infty} f(u)\, g(u - x)\, du$$ $\ni$ $f \star g \Leftrightarrow \bar{F} \cdot G$

$\rightarrow$ $f \star f = \bar{F} \cdot F = |F|^2.$

(*) F. T. In Radio Astronomy.

Periodicity in [Temporal : measured by ν [Hz] : t [s] $\rightarrow$ ν [$\frac{1}{s}$]
Spatial : measured by wavelength λ [m] : m [λ] $\rightarrow$ k [$\frac{1}{m}$]

M87 $\rightarrow$ Sgr A*. — Not consume, inactive

↳ active.

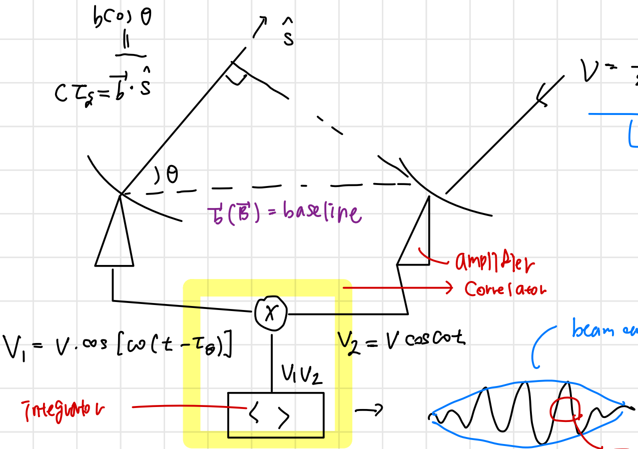
1) dust & gas, can't see in visible light (8% of C)

2) radio ($\lambda = 1.3$ mm)

Interferrometry

- Single dish
  - resolution not enough
  - Tracking ability low
  - Continuum sensitivity restricted ($\nu \lesssim 10$ GHz)

  → Aperture-synthesis interferometer (NZZ)

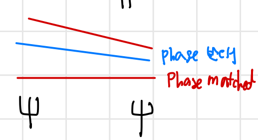


Correlated : multiplied & averaged.

$\nu = \frac{\omega}{2\pi}$, Center frequency

↳ $\Delta\nu \ll \frac{2\pi}{\tau_g}$, $\nu = \frac{\omega}{2\pi}$

$\Delta\theta \sim \frac{\lambda}{B}$



beam center most powerful.

τ_g change : Fringe

$R = \frac{V^2}{2}\cos(\omega\tau_g)$

Flux density

$\frac{V^2}{2} \sim S\cdot\sqrt{A_1A_2}$

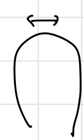
$V_1\cdot V_2 = V^2\cdot\cos\omega(t-\tau_g)\cdot\cos\omega t = \frac{V^2}{2}\cdot[\cos(2\omega t - \omega\tau_g) + \cos\omega\tau_g]$

$$\boxed{R = \langle V_1V_2\rangle = \frac{V^2}{2}\cos\omega\tau_g}$$

∴ Correlator response (Signal of Source)

$$\boxed{R_c = \int I(\hat{s})\cdot\underbrace{\cos\left(2\pi\nu\frac{\vec{b}\cdot\hat{s}}{c}\right)}_{\cos\omega\tau_g} d\Omega = \int I(\hat{s})\cdot\cos\left(2\pi\frac{\vec{b}\cdot\hat{s}}{\lambda}\right)d\Omega}$$

slightly extended.



∴ Slightly extended, spatially incoherent

⇒ Sensitive to symmetric part : I_E. ($I = I_E + I_O$).

⟶ $R_s = \frac{V^2}{2}\sin\omega\tau_g$ 도입 (Sine correlator) ($\frac{\pi}{2}$ phase delay)

$R_s = \int I(\hat{s})\cdot\sin(2\pi\nu\tau_g)\,d\Omega$

∴ Complex Correlator = cos + sin Correlator

$$\boxed{\mathcal{V} \equiv R_c - i R_s = \int I(\hat{s})\exp\left(-i\,2\pi\vec{b}\cdot\hat{s}/\lambda\right)d\Omega} = A\cdot e^{-i\phi}.$$

∴ Complex visibility

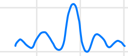
$A = (R_s^2 + R_c^2)^{\frac{1}{2}}$

$\phi = \tan^{-1}(R_s/R_c)$

이제, $\Delta\nu$, Δt 를 확장하자. (유한 bandwidth, time integration)

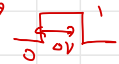
For bandwidth $\Delta\nu$ is small

→ Smearing of fringe.

sinc 

$$\mathcal{V} \equiv \iint_{\nu_c-\Delta\nu/2}^{\nu_c+\Delta\nu/2} I_\nu(\hat{s})\, \exp\left(-i\cdot 2\pi\nu\cdot \tau_g\right) d\nu\, d\Omega \simeq \int I_\nu(\hat{s})\cdot \mathrm{sinc}\left(\Delta\nu\,\tau_g\right) \exp(-i\cdot 2\pi\nu_c \tau_g)\, d\Omega$$

if $I(\hat{s})$ nearly constant : $I_{\nu_c} \simeq I_\nu$: $\mathcal{V} = \int I_{\nu_c}(\hat{s}) \int_{\Delta\nu} e^{-i2\pi\nu\tau_g}\, d\nu\, d\Omega \cdot e^{-2\pi i\nu_c\tau_g}$

⇒  의 $\mathcal{F}(\tau)$

$\Pi\cdot\left(\frac{\nu}{\Delta\nu}\right) = \Delta\nu\, \mathrm{sinc}(\Delta\nu\tau_g)$

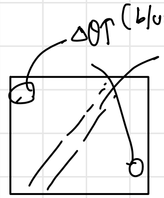
Delay Compensation τ_0 : $\tau_0 \simeq \tau_g = \dfrac{\vec{b}\cdot\hat{s}}{c}$

$c\cdot\tau_g = \vec{b}\cdot\hat{s} = b\cos\theta$

if θ varies (with time) $|c\Delta\tau_g| = b\sin\theta\,\Delta\theta$

$\Delta\nu\Delta\tau_g \ll 1 \rightarrow \Delta\nu \dfrac{b\sin\theta\Delta\theta}{c} \ll 1$ (Fringe 유지 조건)

$\Delta\theta\uparrow$ (b/w) $\Delta\theta\downarrow$



$\theta_s \simeq \dfrac{\lambda}{b\sin\theta}$ → $\boxed{\dfrac{\Delta\theta}{\theta_s} \ll \dfrac{\nu}{\Delta\nu}}$

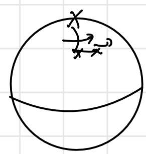
ex) VLA B $\theta_s \simeq \dfrac{\lambda}{b\sin\theta} = \dfrac{0.2\,m}{10^4\,m} = 4$ arcsec.

$\Delta\theta = 900$ arcsec. $\Delta\nu \ll \dfrac{\nu\,\theta_s}{\Delta\theta} = 7\,MHz$.

Also, in case of time. $\Delta\theta$ offset object assume.

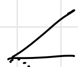
$\omega = \dfrac{2\pi\Delta\theta}{P}$ $(P = 23h\,56m)$ $\rightarrow \phi = \omega\Delta t \ll \theta_s$.

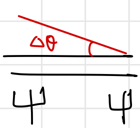
$\dfrac{2\pi}{P}\Delta t \ll \dfrac{\theta_s}{\Delta\theta}$

 ⇒

→ time smearing happens

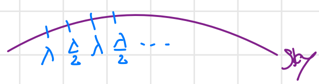
$\Delta\theta = 0$ → North pole reference.



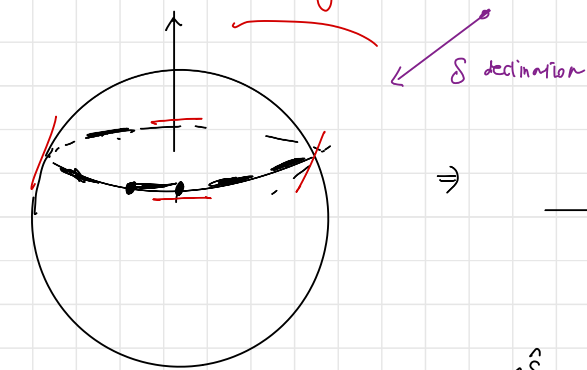


reference offset angle $\Delta\theta$

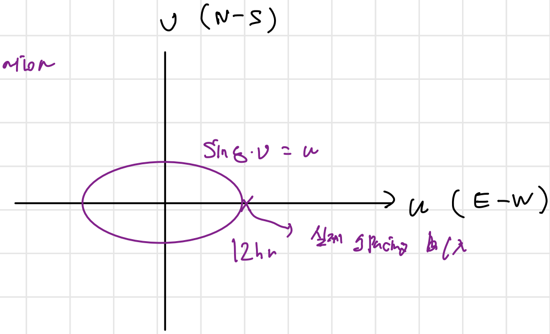
$\Delta\theta \sim \dfrac{\lambda}{B}$. → $\begin{cases} B\downarrow & \Delta\theta\uparrow \text{ (resolve power}\downarrow) \\ B\uparrow & \Delta\theta\downarrow \text{ (resolving power}\uparrow) \end{cases}$

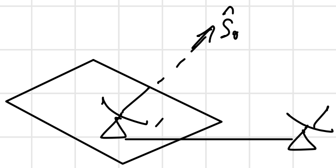


Earth rotation

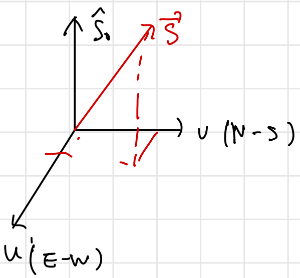


$\Rightarrow$





(*) 3D case



$$\frac{I_\nu(l,m)}{(1-l^2-m^2)^{1/2}} = \iint V(u,v,0)\cdot \exp\left(+i\,2\pi(ul+vm)\right) du\,dv.$$

5. Astrophysic Process

· After Reber (1st radio astronomer), CMB discovery ... $\longrightarrow$ Radio Observation boomed.

$\longrightarrow$ How to interpret?

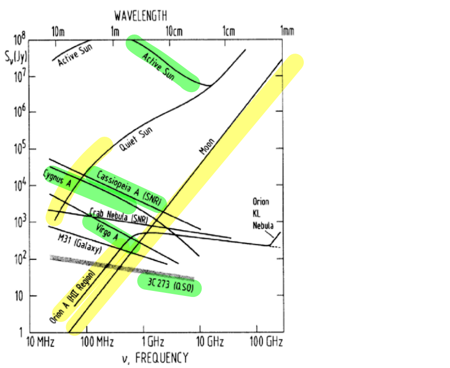


Fig. 10.1 The spectral distributions of various radio sources. The Moon, the quiet Sun and (at lower frequencies) the H II region Orion A are examples of Black Bodies. Close to 300 GHz there is additional emission from dust in the molecular cloud Orion KL. The active Sun, supernova remnants such as Cassiopeia A, the radio galaxies Cygnus A, Virgo A (Messier 87, 3C274) and the Quasi Stellar radio source (QSO) 3C273 are nonthermal emitters. The hatching around the spectrum of 3C273 is meant to indicate rapid time variability. (The 3C catalog is the fundamental list of intense sources at 178 MHz (Bennett 1962))

→ The nature of radio sources

→ Various trend line of astronomical objects in Radio.

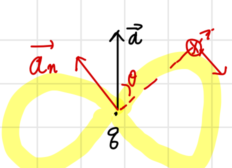
→ What makes these different?

Black body

Nonthermal

1) Bremsstrahlung radiation., motived by highly ionized gas ($T \gtrsim 10^5$ K) ?

① The accelerated charge radiate energy : Larmor formula. (For $v \ll c$, non relativistic)



$E_{tr} \propto \frac{a}{r}\sin\theta$ $\quad (\vec{B} \to B = \frac{E_{tr}}{c})$

$$P(t) = \int_0^{\pi}\int_0^{2\pi} F_r(r,\theta,\phi)\, r^2 \sin\theta \, d\phi\, d\theta$$

$$= \frac{1}{6\pi\varepsilon_0} \cdot \frac{q^2 a^2(t)}{c^3} \quad [W] \longrightarrow \text{Total emitted power}$$

Poynting vector $\vec{F}_p := \frac{\vec{E}\times\vec{B}}{\mu_0}$ $[W \cdot m^{-2}]$.

$$|\vec{F}_p| = \frac{q^2 \sin^2\theta \, a^2(t')}{(4\pi)^2 \varepsilon_0 c^3 r^2}$$

② Now, let's consider the cloud of ionized plasma

To describe the motion of components (Electrons & Ions)

We adapt the Maxwell-Boltzmann distribution

from thermal Equilibrium with non-relativistic ($v \ll c$)

(c.f. $T < 10^9$ K. Above this → Maxwell-Jüttner distribution)

with completely ionized (electron / ions of $+Ze$) plasma

↳ Stationary 가정

$\Rightarrow \langle \frac{1}{2} m v^2 \rangle = \frac{3}{2} k_B T$

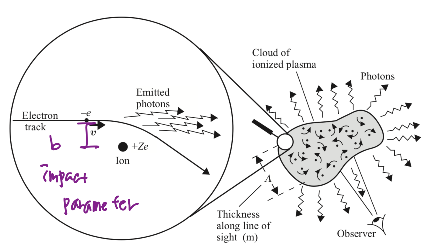


Fig. 5.1: Cloud of plasma (ionized gas) giving rise to photons owing to the near collisions of the electrons and ions. The electrons are accelerated and thus emit radiation in the form of photons. The line-of-sight thickness of the cloud is Λ.

Now, let's check the following

(i) Radiative Energy emitted by a single electron. (Impact parameter b)

Explicitly, $\vec{a} = \frac{\vec{F}}{m} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2}{r^2}\cdot\frac{1}{m}\hat{r} \longrightarrow \boxed{a_{max} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2}{mb^2}}$ ... 1st 근사

Then, $$Q := \int_{-\infty}^{\infty} P(t)\,dt = \frac{1}{6\pi\varepsilon_0}\frac{e^2}{c^3}\int_{-\infty}^{\infty} a^2(t)\,dt \xrightarrow[\Delta t = \tau \sim \frac{b}{v}]{a = a_{max}} \frac{1}{(4\pi\varepsilon_0)^3}\cdot\frac{2}{3}\cdot\frac{Z^2e^6}{c^3m^2b^3v}$$

($\tau_b \sim \frac{b}{v}$; collisional time)

∴ Total E radiated from single electron.

(ii) The frequency of the emitted radiation.

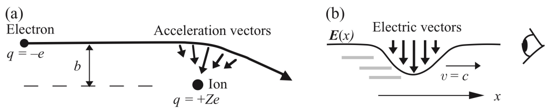


Fig. 5.3: Electron trajectory showing (a) deviation of track, impact parameter b, and acceleration vectors and (b) the pulse of emitted transverse electric vectors for a negative charge. The profile $\boldsymbol{E}(x)$ at a fixed time is shown as a solid line.

$\tau_b \sim \frac{b}{v}$. $\omega \sim \frac{1}{\tau_b} = \frac{v}{b}$

where we define characteristic frequency

$$\nu = \frac{\omega}{2\pi} = \frac{v}{2\pi \cdot b} \rightarrow b = \frac{v}{2\pi\nu}$$

$$\therefore db = -\frac{v}{2\pi\nu^2}d\nu$$

(iii) Thermal electron and a single ion.

The power emitted by *all* the electrons of different speeds that collide with a single proton

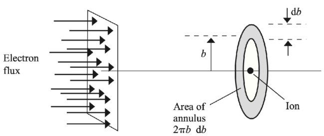


Fig. 5.4: Flux of electrons approaching an ion with the annular region representing the target area at impact radius b in db. The number of electrons that pass through the annuals can be calculated from the flux of electrons and the target area.

Now, let's statistically approach with above situation.

$$\overset{\text{density-like}}{P_b(b,v)\,db} = Q(b,v)\overbrace{\underbrace{[n_e v]}_{\text{Electron flux}}\,2\pi b\,db}^{\text{Electron number/s}}$$

Then, we change the bin

from $db \longrightarrow$ to $d\nu$ (unit frequency)

$$P_b(b,v) \longrightarrow P_\nu(\nu,v)$$

$$\therefore P_\nu(\nu,v) = -P_b(b,v)\frac{db}{d\nu} = \underbrace{\overset{\sim\frac{1}{b^3}}{Q(b,v)}\, n_e v\cdot 2\pi b\,\frac{v}{2\pi\nu^2}}_{\sim\frac{1}{b^2}\sim\nu^2} = \left[\frac{1}{(4\pi\varepsilon_0)^3}\frac{8\pi^2}{3}\cdot n_e\frac{Z^2e^6}{c^3m^2v}\right]\cancel{d\nu}\ [\mathrm{W/ion}]$$

↳ No ν (freq) dependency

→ Rough explaination ⇒ Balance!

(large ν) small b: $a\uparrow$, $Q\uparrow$ but $N\downarrow$ ($A\downarrow$)

(small ν) large b: $a\downarrow$, $Q\downarrow$ but $N\uparrow$ ($A\uparrow$)

Now, let $\frac{1}{2}mv^2 = h\nu_{max} \rightarrow \nu_{max} = \frac{mv^2}{2h}$ (classical limit)

as cutoff frequency.

In the point of view of electron, $\frac{1}{2}mv^2 \geq h\nu$ if we fix ν.

Now, by MB distribution, non degenerate gas follows $P(v)dv = p(\vec{v}) \cdot 4\pi v^2 dv$.

where $p(\vec{v}) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} \cdot \exp\left(-\frac{mv^2}{2k_B T}\right)$

$$\therefore \langle P_\nu(\nu) \rangle_{Ion} = \int_{v_{min} = \sqrt{\frac{2h\nu}{m}}}^{\infty} P_\nu(\nu, v) \cdot p(\vec{v}) \cdot 4\pi v^2 dv \quad [W \cdot Ion^{-1} \cdot Hz^{-1}]$$

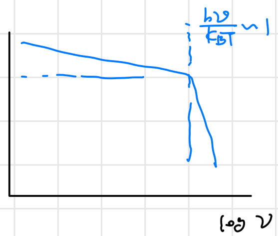
To analyze how it will be observed, let's see spectrum related property.

volume emissivity $j_\nu(\nu)$ $[W \cdot m^{-3} \cdot Hz^{-1}]$

$$j_\nu(\nu) d\nu = n_{Ion} \cdot \langle P_\nu(\nu) \rangle_{Ion} \cdot d\nu \xrightarrow[\int_{v_{min}}^{\infty} dv]{} g(\nu, T, Z) \frac{1}{(4\pi\varepsilon_0)^3} \cdot \frac{32}{3} \cdot \left(\frac{2}{3} \frac{\pi^3}{m^3 k_B}\right)^{\frac{1}{2}} \frac{Z^2 e^6}{c^3} n_e n_i \frac{e^{-h\nu/k_B T}}{T^{1/2}} d\nu$$

$$= C_1\, g(\nu, T, Z)\, Z^2 n_e n_i \frac{e^{-h\nu/k_B T}}{T^{1/2}} d\nu \quad (C_1 = 6.8 \times 10^{-51}\ J \cdot m^3 \cdot K^{1/2})$$

Screening accounting g.



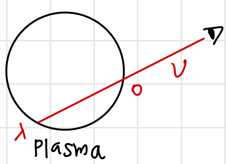
$\nu \uparrow$
$\Rightarrow b \downarrow$
$\Rightarrow$ eventually captured, not anymore "free"

$$j(T) = \int_0^\infty j_\nu(\nu) d\nu = C_2\, \bar{g}(T, Z)\, Z^2 n_e n_i T^{+\frac{1}{2}}$$

$$L(T) = \int_{volume} j(T) dV.$$

(*) Specific intensity $I_\nu(\nu, T)$ $[W \cdot m^{-2} \cdot Sr^{-1} \cdot Hz^{-1}]$

Assume that emission is isotropic. Then,



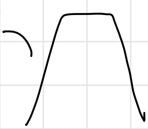
$$I_\nu(\nu, T) = \int_0^\lambda \frac{j_\nu(r, \nu, T)}{4\pi} dr$$

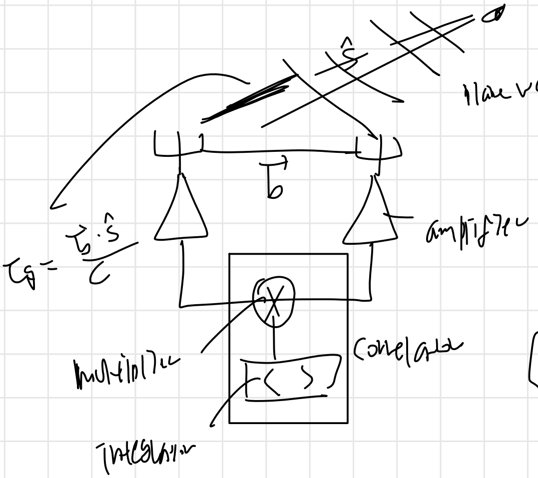
$$= \frac{C_1}{4\pi} g(\nu, T) \frac{e^{-h\nu/k_B T}}{T^{1/2}} \int_0^\lambda n_e^2 dr$$

Emission measure

$$\xrightarrow[\text{If not resolved}]{} S(\nu, T) = \text{Spectral flux density} = \iint I(\nu, T) d\Omega = \frac{1}{4\pi r^2} j_{\nu, av}(\nu, T) \frac{4\pi R^3}{3}$$

self absorption
$\alpha \sim \frac{1}{\nu^2}$





narrow $\Delta\nu \cdot \left(\Delta\nu \ll \frac{2\pi}{\tau_g} \right)$.

$V \sim S_\nu \cdot \sqrt{A_1 A_2}$,

$$\begin{cases} V_1 = V_0 \cos \omega(t-\tau) \\ V_2 = V_0 \cos \omega t. \end{cases}$$

$$V = V_1 V_2 = V_0^2 \cdot \cos \omega(t-\tau) \cos \omega t.$$

$$= \frac{V_0^2}{2} \left[\cos \omega(2t-\tau) + \cos \omega\tau \right] \xrightarrow{\text{int}} \frac{V_0^2}{2} \cos \omega\tau.$$

Sin Correlator $\longrightarrow \frac{V_0^2}{2} \cdot e^{-i\omega\tau}$ $\qquad \omega\tau = 2\pi\nu \frac{\vec{b}\cdot\hat{s}}{c} = 2\pi \cdot \frac{\vec{b}}{\lambda} \cdot \hat{s}$

$$\text{Visibility} = \int I(\hat{s}) A_e(\hat{s}) \cdot e^{-i 2\pi \cdot \frac{\vec{b}}{\lambda} \cdot \hat{s}} \, d\Omega.$$

$$= \int I(\hat{s}) A_e(\hat{s}) \exp\left(-2\pi i \cdot \left(ul + vm + \cancel{w\sqrt{1-l^2-m^2}}\right)\right) \frac{dl\,dm}{\cancel{\sqrt{1-l^2-m^2}}}$$

$$V(u,v,0) = \int I \cdot A \cdot \exp\left(-2\pi i \,(ul+vm)\right) dl\,dm$$

$w \approx 0$

$\Big\downarrow \mathcal{F}^{-1}$

$I \cdot A.$